CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

MAY/JUNE 2008

PURE MATHEMATICS
(Rest of the Region)

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
MAY/JUNE 2008

INTRODUCTION

This is the first year that the revised syllabus for Pure Mathematics is examined. The new format of Paper 01 is multiple choice (MC) and Papers 02 and 03 have retained the format with extended-response questions.

Generally, the performance of candidates was very satisfactory with a small number of excellent to very good grades. There still remained, however, too large a number of weak candidates who seemed unprepared for the examination. Approximately 3700 scripts were marked.

GENERAL COMMENTS

UNIT 1

The new topics in the revised Unit 1 syllabus are Cubic Equations, Indices and Logarithms, and L'Hopital's rule, with Complex Numbers moved to Unit 2. Of these new topics, candidates showed reasonable competence in Cubic Equations and Logarithms, but some seemed not to have been exposed to L'Hopital's rule. Among the old topics comprising Unit 1, candidates continue to experience difficulties with Indices, Mathematical Induction and Summation Notation ($\sum$). General skills at algebraic manipulation including substitution at all levels continue to pose challenges. A new area of difficulty has emerged, the topic of Trigonometric Identities. Strong performances were recorded in Differentiation, the Plotting of Graphs, Vectors, and Coordinate Geometry. This was encouraging. Some effort should be made in providing students with practice in connecting parts of the same question in order to facilitate efficient solutions.

DETAILED COMMENTS

UNIT 1
PAPER 01

Paper 01 comprised 45 multiple-choice items. The candidates performed satisfactorily. The mean score was 58 per cent with a standard deviation of 7.7.

UNIT 1
PAPER 02

Question 1

Specific Objectives(s): (a) 1, 2, 3, 5, 7, 8; (b) 1, 3, 4, 6; (f) 5 (ii)

The question tested properties of cubic equations, surds and the process of summation.

Overall, the question was attempted by over 90 per cent of the candidates. Various approaches were used in each part with a high degree of success.

(a) Few mistakes were made in this part; however, incorrect expansions of $(x + 1)\ (x - 1)\ (x - 3)$ were mainly responsible for those that occurred. Incorrect equating of coefficients was also a main source of error.

(b) This part of the question was generally well done, however, some candidates had some difficulties in rationalizing the surds. Many gained full marks in (ii) by using the result in (i).
Errors occurred in (ii) by incorrectly evaluating$\sqrt{12}$.

(c) (i) Many candidates used the principle of mathematical induction to obtain the result. Some candidates, in using this method, had difficulty manipulating the algebra involved. Others, who observed that results on the formula sheet were appropriate, had an easier passage to the final result.

(ii) Candidates were not as successful in this part as in (i). The most frequent error occurred in the separation of the summation

$$\sum_{r=31}^{50} r(r+1) = \sum_{r=1}^{50} r(r+1) - \sum_{r=1}^{30} r(r+1).$$

A common error was subtracting

$$\sum_{r=1}^{31} r(r+1) \text{ instead of } \sum_{r=1}^{30} r(r+1).$$

Answer(s): (a) $p = -1,\ q = -1,\ r = 3$

(c) (ii)

$$\sum_{r=31}^{50} r(r+1) = 34\ 280$$

Question 2

Specific Objective(s): (f) 1, 5 (i); (c) 1, 3 (i), (ii).

This question tested quadratic equations, the sums and products of the roots of such equations, the solutions of quadratic equations and logarithms.

(a) This part of the question dealt with quadratic equations. Approximately 99 per cent of the candidates attempted this question and several obtained at least 10 of the 12 marks allocated to this part.

(b) There was mixed success with this part of this question which examined logarithms in (ii) and (iii). Many candidates did (i) successfully although there were some who did not obtain the correct value for x from $x^{1/3} = -1$. In Part (ii), some candidates did not discard the negative value of x while in (iii) some candidates unwisely used calculators although the question stated that calculators should not have been used.

Answer(s): (a) (i) $\alpha + \beta = -2,\ \alpha\beta = 5/2$

(ii) a) $\alpha^2 + \beta^2 = -1$

b) $\alpha^3 + \beta^3 = 7$

(iii) $8x^2 - 56x + 125 = 0$

(b) (i) $x = 64,\ -1$

(ii) $x = 2$

(iii) -1

UNIT 1
PAPER 02
SECTION B
(Module 2: Trigonometry and Plane Geometry)

Question 3

Specific Objective(s): (a) 4, 5, 9, 12; (b) 1.

This question tested the candidates' ability to use the gradients of line segments and to develop and apply trigonometric identities.

This question was not popular with most of the candidates. Most candidates attempting this question scored in the range of (0 - 4) marks, which was very disappointing.

(a) The majority of the candidates did not recognize the relationship between the gradient of the straight line and the tangent of the angle between the line and the positive direction of the x-axis. These candidates did not link the word "tangent" with "tan", finding instead points and lines (in many variations). The few who recognized "tangent of an angle", frequently found "$\tan\alpha - \tan\beta$" instead of "$\tan(\alpha - \beta)$" where $\alpha > \beta$. Some of those who started well seemed not to be aware that $\tan(\alpha \pm \beta) \neq \tan\alpha \pm \tan\beta$, which spoiled the work thereafter.

(b) The majority of candidates attempted (i) and were capable of finding the correct identities to replace $\sin 2\theta, \cos 2\theta$ *and* $\tan\theta$, but some had difficulty manipulating the identity to get to $\tan\theta$ in terms of $\sin 2\theta$ *and* $\cos 2\theta$.

In (ii) a significant number of candidates did not realize that "Express $\tan\theta$ in terms of $\sin 2\theta$ *and* $\cos 2\theta$" meant change the subject (or transpose) the formula given in (i). The word "hence" was not understood by the candidates to use previous work and thus in most cases this was not done.

Several **errors** were made in (iii) such as: $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$ $\quad$ $\cos 2\theta = \frac{1-\tan^2\theta}{1+tan^2\theta}$

$\tan\theta = \frac{\sin 2\theta}{\cos 2\theta}$ $\quad$ $\cos\theta = 1 - \sin\theta$

(c) A large number of candidates did not use the fact that the angles of a triangle add up to 180^0. Instead, it was stated that A + B = C or in some cases, particular values of angles were used. Therefore, they did not recognize that the sine of an angle is equal to the cosine of its complement. In (c) (i) b), those candidates who actually attempted the question chose the correct factor formula but were unable to follow through for the second mark. Very few of the candidates recognised the link between (c) (i) b) and (ii). They did not recognise that A could be used as a double angle so they failed to use $\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$.

Answer(s): (a) (i) $\tan\alpha = 3;\ \tan\beta = \frac{3}{4}$

(ii) $\tan(\alpha - \beta) = \frac{9}{13}$

(b) (ii) $\tan\theta = \frac{\sin 2\theta}{1+\cos 2\theta}$

Question 4

Specific Objective(s): (b) 1, 2, 3, 5, 7, 8.

This question examined the application of coordinate geometry to the properties of a circle, straight lines, tangents, normal, and intersections between straight lines and curves. For this question, Part (b) proved more challenging to the candidates than Part (a).

Part 4 (a) (i) was fairly well done. Seventy per cent of the candidates attempted the question and were able to get most of the eight marks allocated to it. The remaining 30 per cent of the candidates had difficulties with

(a) finding the midpoint of a line
(b) calculating the gradient of a line using the formula
(c) knowing that the perpendicular bisector passes through the midpoint
(d) knowing that the gradient of the perpendicular bisector is the negative reciprocal of the gradient of the line.

(ii) Of the few candidates that actually attempted this question, most were unable to recognize that the centre of the circle was the point of the intersection of the perpendicular bisector of any two chords of the circle. The alternative solution of substituting the points into the equation of the circle was also challenging for some candidates. Candidates substituted incorrect values and could not solve simultaneously the equations generated.

The main areas of concern in the students' approach were as follows:

(a) Some candidates could not write coordinates properly as (x, y).

(b) Some candidates were using the formula for finding the length of a line to calculate the midpoint of the line.

(c) When calculating the equation of the perpendicular line, several candidates erroneously submitted either the point P (-2, 0) or Q (8, 8), even when the midpoint was correctly found.

Most of the candidates solved Part 4 (b) (i) correctly. However, many of them did not recognise the significance of the repeated roots in relation to the tangent of the circle.

Some candidates also correctly used alternative solutions such as

(a) finding the perpendicular distance from the centre of the circle by means of the formula $\frac{|ax+by+c|}{\sqrt{a^2+b^2}}$ and showing that this is equal to the radius of the circle

(b) showing that the gradient of the line from the centre of the circle to the tangent is the negative reciprocal of the gradient of the tangent and therefore the line and tangent are perpendicular to each other.

Part 4 (b) (ii) was very well done. In fact, most candidates were able to recover from (b)(i) as above.

Answer(s): (a) (i) $4y + 5x = 31$

(ii) $x = -1,\ y = 9$

(b) (ii) $x = 0,\ y = 1$

SECTION C
(Module 3: Calculus 1)

Question 5

Specific Objective(s): (a) 5, (b) 5, 6, 11-19

The question tested knowledge of limits, differentiation and integration. Curve sketching and the nature of turning points of a curve were also investigated.

(a) This part of the question dealt with the limit of a rational function in which both numerator and denominator are polynomials.

Candidates showed knowledge of the methods involved in finding the required limit but fell down on the mechanics of factorizing the polynomials correctly. The main difficulty occurred in the factorization of $x^3 - 27$. Many candidates stated that $x^3 - 27 = (x-3)(x^2-9)$.

(b) Very few candidates performed well on this part of the question. The main area of weakness was in differentiating the term $\frac{u}{t}$. As a consequence, many candidates had difficulty in deriving the appropriate equations required for the correct solutions.

(c) There were mixed performances on this part of the question. In finding the equation for y, many candidates omitted the constant of integration when integrating $\frac{dy}{dx}$, and this led to an incorrect equation for the curve C and an incorrect sketch. In spite of this, Part (ii) was reasonably well done.

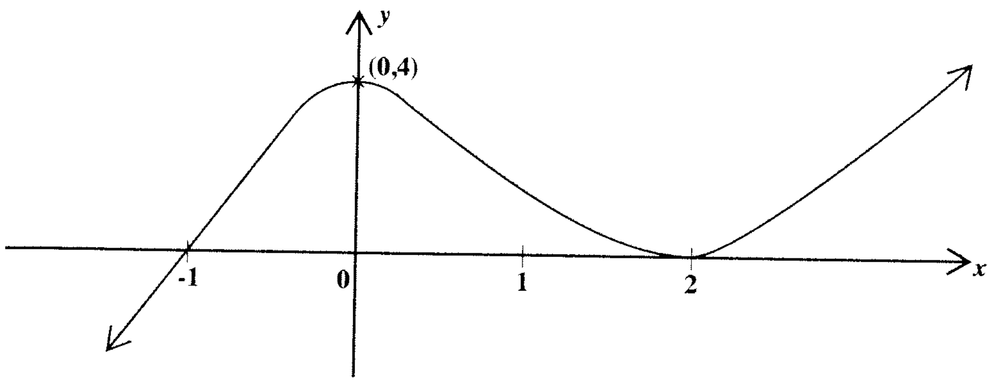
Answer(s): (a) $\frac{27}{7}$

(b) $u = \frac{4}{5}$, $v = \frac{-9}{5}$

(c) (i) $y = x^3 - 3x^2 + 4$

(ii) Stationary points are (0,4) and (2,0).
(0,4) is a maximum and (2,0) is a minimum.

(iii)



Question 6

Specific Objective(s): (b) 5, 6, 8, 9, 10 (c) 4, 6, 8 (i)

The question tested aspects of differentiation, integration and some applications to mensuration. The question was poorly done.

(a) This part of the question covers basic differentiation. In both parts, many candidates found difficulty in applying the chain rule to the process of differentiation. In too many cases, the function in (i) was replaced with $(2x^2 - x)^{1/2}$ or $x\left(2x^{1/2} - 1^{1/2}\right)$. In Part (ii), $\sin^2(x^3 + 4)$ was not interpreted correctly as a function requiring the chain rule for differentiation and terms were lost in the process.

(b) (i) Generally, candidates did not appear to recognize the need to apply the linearity property of integrals.

(ii) For those candidates who recognized the need to integrate the given function between limits 1 and 3, a few were unable to follow through to obtain the solution of k = 4. Many were unable to manipulate the fractions after substitution of the limits. Several candidates simply substituted the limits into the given function without integrating to find the required area.

(c) (i) Many candidates mistakenly used the volume of the sphere rather than that of the hemisphere.

(ii) Several candidates neglected to include the base area when finding the total surface area.

(iii) Of those candidates who answered this part of the question, many did not verify that A was indeed a minimum when r = 3.

Answer(s): (a) (i) $\frac{3x-1}{\sqrt{(2x-1)}}$

(ii) $6x^2 \sin(x^3 + 4)\cos(x^3 + 4)$

(b) (i) 3

(ii) $k = 4$

(c) (ii) r = 3

UNIT 1
PAPER 03/B (ALTERNATE TO INTERNAL ASSESSMENT)
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

Specific Objective(s): (c) 1, 2, 3, 4; (d) 7; (f) 5 (i)

The question tested properties of quadratic equations and their roots, functions, indices and logarithms.

(a) There were several good attempts at this part of the question but few candidates obtained complete solutions.

(b) (i) This part was poorly done. Many candidates did not observe that a critical approach to the solution was to start with $f(0) = 6$. Others did not discern that substituting $x = 3$ in the given equation would provide a lead to obtaining $f(9)$.

(ii) This part was fairly well done.

(c) Both Parts (i) and (ii) were well done.

Answers: (a) (i) Roots are -3, -9

(ii) k = 27

(b) (i) $f(3) = 15 \text{ and } f(9) = 33$

(ii) $x = 6$

(c) (i) 30

(ii) 78

UNIT 1
PAPER 3 B
SECTION B
(Module 2: Sequences, Series and Approximations)

Question 2

Specific Objective(s): (a) 5, 13; (c) 7, 8, 9, 10.

This question tested candidates' ability to solve trigonometric equations for a given range; to apply the properties of vectors; to find the scalar (dot) product and the angle between the given vectors as well as the magnitude and direction of a vector.

(a) Candidates did not display sufficient knowledge of the trigonometric identities. The solutions presented were incomplete. Overall, this part of the question was poorly done.

(b) In most cases, candidates recognized the need to use the dot product, but some of them did not know that **a . b = 0** for perpendicularity.

(c) While a number of candidates knew the formula to find the acute angle between the two vectors, few knew how to manipulate it correctly.

(d) Candidates were able to evaluate correctly the magnitude of F in most of the cases. Few correctly attempted to evaluate the angle of inclination requested in Part (b).

Answer(s): 2 (a) $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

(b) (i) $t = \frac{12}{5}$

(ii) $\theta = 22.3^0$

(iii) a) $|F| = 3.60$

b) $\emptyset = 56.3^0$

SECTION C
(Module 3: Calculus I)

Question 3

Specific Objective(s): (b) 5, 6, 7(i), 16, 21; (c) 1, 2, 3, 4, 5(i), (ii), 9

3. The question tested knowledge of the differential and integral calculus, the equation of a normal to a curve and rate of change in calculus.

   (a) This part of the question was reasonably well done although a small number of the candidates experienced difficulties in differentiating the term $\frac{1}{x}$ in $y = x + \frac{1}{x}$.

   (b) Some candidates had problems with expressing the integrand as a sum of three separate terms. Very few candidates obtained full marks for this part.

   (c) Most candidates obtained a differential equation in V without the negative sign. Nevertheless a good understanding of the concept involved was exhibited in the solutions.

Answer(s): (a) (ii) $3y + 4x = 31$

(b) $-\frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} + a\ constant$

(c) (i) $\frac{dV}{dt} = -30t$

(ii) Liquid lost is $75\ cm^3$.

GENERAL COMMENTS

UNIT 2

In general, the performance of candidates in Unit 2 may be regarded as satisfactory. A small number of candidates reached an outstanding level of proficiency. A number of candidates were inadequately prepared for the examinations.

Topics in Calculus, Simple Probability, Approximations to Roots of Equations, and Series were satisfactorily covered. The examination tested new topics which included Calculus of Inverse Trigonometrical Functions and the Second Derivative, the use of an Integrating Factor for First-order Differential Equations, Second-order Differential Equations, Maclaurin's Theorem for Series Expansions, Binomial Expansion Series for Rational and Negative Indices, Complex Numbers, De Moivre's Theorem for integral n, and the Locus of a Complex Number.

Weaknesses in algebraic manipulation and tasks involving substitution were manifestly evident. Candidates obviously found it difficult to solve problems which required these applications. It is imperative that more emphases be placed on these areas of weakness. Extensive practice in the use of substitution and algebraic manipulation is necessary if candidates are to be well-prepared to show improved performances in these topics. Candidates continue to demonstrate a lack of appreciation for questions which allow for "hence or otherwise". They fail to see existing links from previous parts of a question, and rarely seem disposed to using "otherwise," thus employing any other suitable method for solving the particular problem.

PAPER 01

Paper 01 comprised 45 multiple-choice items. The candidates performed fairly well with a mean score of 64 per cent and standard deviation of 8.6.

DETAILED COMMENTS

UNIT 2
PAPER 02
SECTION A
(Module 1: Calculus II)

Question 1

Specific Objective(s): (a) 6, (b) 2, 5 (c) 1, 5, 6

This question tested the differentiation of exponential, trigonometric and logarithmic functions, resolution into partial fractions, and integration involving inverse trigonometric functions.

All candidates attempted this question with varying degrees of success.

(a) (i) Most of the candidates omitted the constant π in the derivative. Answers included in part, $e^{4x} \sin \pi x$ ...
A majority of the candidates treated e^{4x} as a constant rather than as a function of x. Consequently, some candidates did not obtain $4e^{4x}$ as the derivative of e^{4x}. Emphases must be placed on recognizing functions of a stated variable as against constants.

(ii) The majority of candidates attempting this question opted to use the chain rule $\frac{d}{du}\ln(u) \times \frac{d}{dx}\left[\frac{x^2+1}{\sqrt{x}}\right]$. However, they failed to apply the quotient rule correctly for $\frac{d}{dx}\left[\frac{x^2+1}{\sqrt{x}}\right]$, and were unable to secure full marks for this part of the question.

(b) Many candidates expressed y as $\frac{1}{3^x}$ and attempted to use the quotient rule which, for some, ran into difficulties. The majority of candidates used $\log_{10}$ or $\log_3$ and attempted to differentiate with respect to x. Some candidates **erroneously** expressed $y = \ln_3 x$ and attempted to differentiate with respect to x. A small number of candidates **erroneously** stated $\ln y = \ln 3^{-x}$ giving $-x = \frac{\ln y}{\ln 3}$.

It is clear that this method of differentiation was new to many candidates. Logarithmic and implicit differentiation were not part of many of the candidates' skills.

(c) (i) The majority of candidates demonstrated competence in this topic. Some errors in simple arithmetic were common. A few candidates had problems simplifying the terms of the fractions, thus making it difficult to answer Part (ii) successfully.

(ii) The majority of candidates were successful in obtaining the correct partial fractions from Part (i). However, the evaluation of

$$\ldots \int \frac{-5}{2(x^2+1)}\,dx$$

proved challenging for almost all the candidates. It seems that insufficient tutorials and practice in inverse trigonometrical calculus contributed to candidates' inability to complete the integration to this part of the question.

Candidates continue to omit the constant of integration from indefinite integrals. This results in loss of marks, and impacts negatively on their overall performances. The concept of the constant of integration must be fully explained so that candidates can be aware of the importance of including it in their resulting integrals.

Answer(s): (a) (i) $e^{4x}(4\cos\pi x - \pi\sin\pi x)$

(ii) $\frac{2x}{x^2+1} - \frac{1}{2x} = \frac{3x^2-1}{2x(x^2+1)}$

(c) (i) $\frac{3}{2(x-1)} + \frac{x-5}{2(x^2+1)}$

(ii) $\frac{3}{2}\ln|x-1| + \frac{1}{4}\ln|x^2+1| - \frac{5}{2}\arctan x + \text{const}$

Question 2

Specific Objective(s): (c) 5, 7, 8, 11.

This question required candidates to use an integrating factor, integration of exponential functions, and integration by parts for a definite integral.

The majority of candidates attempted this question. However, many of them were unable to secure maximum marks on all parts of the question for various reasons.

(a) The majority of candidates recognized the use of an integrating factor. However, many of them failed to evaluate this factor correctly. In addition, having found the integrating factor, many of them did not multiply the equation by the integrating factor. As a result, these candidates were not able to solve the differential equation completely.

(b) This question was satisfactorily done by most of the candidates who earned the maximum mark.

(c) The responses to this question revealed some weaknesses in identifying which of the terms to take as v and which as $\frac{du}{dx}$. This resulted in many candidates having to evaluate $\int \ln x\, dx$, since x^2 was taken as v. Those candidates who integrated correctly had some difficulty evaluating the integral using the stated limits. In general, candidates demonstrated a satisfactory understanding of integration by parts.

(d) (i) Many candidates continue to show weakness in working with given substitutions. Many of them failed to find $-dv = du$. They, in fact, substituted

$$v = 1 - u \text{ to get } \int \frac{1}{\sqrt{v}} du$$

Common **errors** included $\int -v^{\frac{-1}{2}}$, omitting dv, not stating the constant of integration, and not replacing $v^{\frac{1}{2}}$ with $\sqrt{1-u}$ in the final answer. **The use of substitution for integration must be extensively practised in order for candidates to show improved performances in this topic.**

(ii) The majority of candidates were able to replace $\cos x\, dx$ with du. However, the substitution $u = \sin x$ was not correctly used to transform the original integral into a manageable form using the given substitution. Candidates were unable to express $\sqrt{(1+\sin x)}$ in terms of u. As a result they were unable to obtain the cancellation of the term $\sqrt{(1+u)}$ to obtain $\frac{1}{\sqrt{1-u}}$ in order to make use of the answer at (i). Candidates were unable to obtain maximum marks for this part of the question.

Answer(s): (a) $y = \frac{1}{3}e^{2x} + \frac{c}{e^x}$

(b) $4y = e^{4x} + 3$

(c) $\frac{1}{9}(2e^3 + 1)$

(d) (i) $I = -2\sqrt{(1-u)} + C$

(ii) 2

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 3

Specific Objectives: (a) 2, 5, 12, (b) 5, 7, 9, 10, (c) 13.

This question tested the candidates' abilities to use the recurrence relation of a sequence, apply the principle of mathematical induction, geometric progression, and the application of Maclaurin's expansion series.

(a) (i) This part of the question was well done by the majority of candidates. Simple substitution was required.

(ii) Few candidates demonstrated a sound understanding of the principle of mathematical induction. Moreover, the majority of candidates are still unclear about the inductive process and failed to show proper proof that the statement P_k and P_k+1 were true for n = some k. More work on the principle of mathematical induction is required. Some candidates simply substituted $k + 1$ for k in the statement for P_k.

(b) The majority of candidates were able to obtain the equations in terms of a and r for the given conditions. However, weaknesses in algebra continue to be evident and many students failed to solve for a and r correctly. Few candidates gained full marks for this part of the question.

(c) (i), (ii), (iii) This part of the question was poorly done. Insufficient exposure and practice in using Maclaurin's theorem for expansions were evident. The Maclaurin's theorem is given in the Formulae Booklet issued to candidates at examinations and should have made it easier for them to answer this part of the question. Candidates also omitted the range of values of x for which the expansions are valid. This is one of the additional topics tested for the first time and it is evident that much more needs to be done by way of tutorials and practice.

Answer(s): (a) (i) 3, 4, 6, 9

(b) $a = 27,\ r = \frac{2}{3}$

(c) (i) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \ldots \qquad -1 < x \leq 1$

(ii) (a) $-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \ldots \qquad -1 \leq x < 1$

Question 4

Specific Objective(s): (b) 13, (c) 3, 4, (e) 1, 2, 4.

This question tested the concept of existence of roots for a continuous function, the Newton-Raphson method of approximation, the binomial expansion series for positive and negative rational indices, Maclaurin's theorem, and use of expansion series for calculating the fractional value of a surd.

(a) (i) The majority of candidates knew the principle of the Intermediate Value Theorem. However, very few of them stated that the function was a polynomial and more importantly that it was continuous.

(ii) This part of the question was well done. Some students misinterpreted the rubric and proceeded to find a numerical value for the root α.

(b) (i) All of the candidates opted to use the binomial expansion series for this part of the question. Arithmetical errors in calculating the coefficients of terms resulted in candidates losing marks. Very few students stated the range of values of x for which the expansion is valid. Emphasis should be placed on this aspect.

(ii) No candidate was able to deduce this expansion from (i) and proceeded to use the binomial expansion. As in (i), arithmetical errors, particularly signs of the coefficients, resulted in loss of marks. Candidates omitted the range of values of x.

(iii) Candidates who were able to complete (i) and (ii) correctly gained full marks on this part of the question.

(iv) Most candidates had difficulties using the given substitution and in reducing the value of $\sqrt{2}$ to the required answer. Surds continue to be challenging for many candidates. They have difficulties working numerical problems without the use of calculators or tables.

Answer(s): (b) (i) $1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \ldots, \quad -1 < x < 1$

(ii) $1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16} + \ldots, \quad -1 < x < 1$

SECTION C

(Module 3: Counting, Matrices and Complex Numbers)

Question 5

Specific Objective(s): (a) 2, 3, 4, 6, (b) 1, 2, 7, 8.

This question tested the principle of combinations, matrix algebra, and complex numbers.

(a) (i) Responses to this part of the question were very satisfactory. Some candidates had difficulties distinguishing between combinations and permutations.

(ii) The majority of candidates gained full marks for this part of the question, having obtained (i) correctly.

(b) (i) (a) Generally, this part of the question was well done. Errors were mostly due to incorrect arithmetic.

(b) Candidates demonstrated good techniques for multiplying conformable matrices.

(ii) A significant number of candidates were not able to deduce A^{-1} from the previous results. Many of them attempted to find the inverse of A by the process $\frac{1}{|A|}$ adj A. Arithmetic errors did not allow some of them to obtain the correct answer. Some candidates also attempted to use row reduction of the augmented matrix $\left(\begin{array}{ccc|ccc} 3 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{array}\right)$ but had difficulties completing the process. A number of candidates were able to deduce A^{-1} form (i) b).

(iii) Most candidates demonstrated weaknesses in matrix algebra for this part of the question. Instances were seen where candidates merely stated $X = \frac{A-B}{A}$.

Answer(s): (a) (i) 70

(ii) 65

(b) (i) a) $\begin{pmatrix} 2 & 2 & -1 \\ 1 & 0 & 2 \\ 1 & -4 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

(ii) $A^{-1} = \frac{1}{3}M = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & -1 \\ \frac{-1}{3} & 1 & \frac{-1}{3} \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & \frac{-2}{3} & 0 \\ -1 & 4 & -1 \\ 0 & \frac{2}{3} & 2 \end{pmatrix}$

Question 6

Specific Objective(s): (c) 4, 7, 8, 9, 10.

This question tested rationalization of a complex number, determining the value of a multiple of a complex number, use of a conjugate, and determining the loci of a complex number.

(a) (i) This part of the question was well done.

(ii) Arithmetical errors resulted in a few candidates stating an incorrect value for λ.

(iii) Candidates used more than one method to answer this part of the question. With (i) and (ii) some candidates used the binomial expansion. Some candidates used De Moivre's theorem. Generally this part of the question was well done.

(b) (i) Apart from some difficulties with algebra, candidates performed well in this part of the question.

(ii) Approximately 95 per cent of the candidates did not attempt this question. Candidates failed to see the link with (i) and were unable to determine the technique to be used. Seemingly, the area relating to loci of complex numbers was not extensively dealt with. Candidates should be exposed to much more of this, with adequate practice.

Answer(s): (a) (i) $\frac{1}{2}(1 - i)$

(ii) $\frac{1}{2}$

(iii) $\frac{-1}{4}$

(b) (ii) $C\left(\frac{1}{6}, 0\right)$

UNIT 2
PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)
SECTION A
(Module 1: Calculus II)

Question 1

Specific Objective(s): (b) 2, 3, 5.

This question examined parametric differentiation and its application to a normal to a curve, and a modelling based on exponentials, differential equations, rate of change, and a graph of the model.

(a) (i) Most of the candidates who attempted this part of the question were unable to determine $\frac{dy}{dt}$ and $\frac{dx}{dt}$, hence could not evaluate $\frac{dy}{dx}$. In addition the value for x at $y = 18$ was not found.

(ii) Consequently, the correct equation of the normal was not determined. A majority of candidates did not apply the gradient of the normal as $\frac{-dx}{dy}$.

(b) (i) A majority of the candidates did not attempt this part of the question. The few who did demonstrated a lack of understanding of differentiation of natural logarithms.

(ii) a) This part of the question was poorly done since there was no follow-through from (i) to work with.

b) Candidates who attempted this part of the question substituted $180 = \frac{dV}{dt}$ instead of $V = 180$.

(iii) Only a few candidates attempted this part of the question. No candidate obtained full marks. Errors included no labels on axes and failing to use the fact that $t \geq 0$.

Overall this question was poorly done.

Answer(s): (a) (i) $\frac{1}{3}$

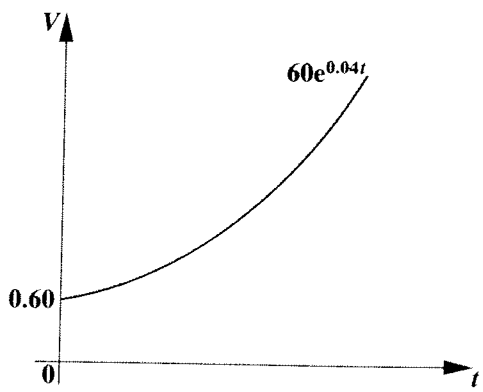
(ii) $3x + y - 99 = 0$

(b) (i) $2.4e^{0.04t}$

(ii) (a) 3.58

(b) 7.2

(iii)



Question 2

Specific Objective(s): (a) 2, 3, (b) 5, 8, 9, (c) 3, 4.

This question tested the arithmetic and the geometric progressions, the binomial expansion series for a negative index, and the Maclaurin's theorem for expansion of a trigonometric function.

(a) (i) a) Generally, this part of the question was poorly done. Candidates could not determine the geometric progression.

b) There was no follow-through after being unable to obtain a).

(ii) There was no follow-through after being unable to obtain the previous results.

(b) (i) (ii) Responses to these parts of the question were poor.

(c) (i) Some candidates attempted this part of the question but obtained the wrong answer due to arithmetical errors.

(ii) Candidates could not use the expansion given to express $\sec x \ as \ \frac{1}{\cos x}$ algebraically. No candidate attempted differentiation to obtain the coefficients for the expansion of sec x.

Answer(s): (a) (i) a) $5 \times 2^{r-1}$

b) $5(2^n - 1)$

(ii) $n = 8$

(b) (i) $S = (1 + 3 + 5 + 7 + \ldots) + \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots\right)$

(ii) $n^2 + 1 - \left(\frac{1}{2}\right)^n$

(c) (i) $1 + y + y^2 + y^3 + y^4 + \ldots$

(d) $1 + \frac{x^2}{2} + \frac{5x^4}{24} + \ldots$

Question 3

Specific Objective(s): (a) 8, 9, 11, 12, 13; (b) 1, 2, 5, 6.

This question tested matrix algebra, and classical probability.

(a) (i) Most candidates were able to state the augmented matrix. However, beyond this point there were serious challenges. They were unable to perform the necessary row reduction.

(ii) No candidate answered this part of the question.

(b) (i) Most of the candidates re-stated the probabilities given. No evidence of the use of the Venn diagram or the formula for solving the required probability was seen.

(ii) a) b) Responses to these parts of the questions were poor. Candidates seemed not to know the definitions of independent and mutually exclusive events.

Answer(s): (a) (i) -1

(ii) $x = 5 - 23z,\ y = 6z - 1$

(b) (i) 0.05

(ii) a) $P(A \cup B) \neq P(A) + P(B)$; not independent

b) $P(A \cup B) \neq 0$; not mutually exclusive

PAPER 03 – INTERNAL ASSESSMENT

The Internal Assessment comprises these Module tests.

The main features assessed are the:

- Mapping of the items tested to the specific objectives in the syllabus for the relevant Unit
- Content coverage of each Module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (Module tests and students' scripts)
- Quality of the teachers' solutions and mark schemes
- Quality of the teachers' assessments – consistency of marking using the mark schemes
- Inclusion of mathematical modelling in at least one Module test for each Unit

GENERAL COMMENTS

1. Too many of the Module tests comprised items from CAPE past examination papers.

2. Untidy 'cut and paste' presentations with varying font sizes were common place.

3. This year there was a general improvement in the creativity of the items, especially with regards to mathematical modelling. Teachers are reminded that the CAPE past examination papers should be used ONLY as a guide.

4. The stipulated time for Module tests (1-1$^1/_2$ hours) must be strictly adhered to as students may be at an undue disadvantage when Module tests are too extensive or insufficient. The following guide can be used: 1 minute per mark. About 75 per cent of the syllabus should be tested and mathematical modelling **MUST** be included.

5. Multiple-choice Questions will **NOT** be accepted in the Module tests.

6. Cases were noted where teachers were unfamiliar with recent syllabus changes, for example, complex numbers, 3-dimensional vectors, dividing a line segment internally or externally.

7. The moderation process relies on the validity of teacher assessment. There were a few cases where students' solutions were replicas of the teachers' solutions – some contained identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on the students' scripts did not correspond to the marks on the Moderation sheet.

8. Teachers **MUST** present evidence of having marked each individual question on the students' scripts before a total is calculated at the top of the script. The corresponding whole number score out of 20 should be placed at the front of the students' scripts.

9. To enhance the quality of the design of the Module tests, the validity of teacher assessment and the validity of the moderation process, the Internal Assessment guidelines are listed below for emphasis.

Module Tests

(i) Design a separate test for each Module. The Module test MUST focus on objectives from that module.

(ii) In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered, and a common marking scheme used. One sample of FIVE students will form the sample for the centre.

(iii) In 2009, the format of the Internal Assessment remains unchanged.
[Multiple Choice Examinations will NOT be accepted.]

GUIDELINES FOR MODULE TESTS AND PRESENTATION OF SAMPLES

1. COVER PAGE TO ACCOMPANY EACH MODULE TEST

The following information is required on the cover of each Module test.

- Name of School and Territory, Name of Teacher, Centre Number.
- Unit Number and Module Number.
- Date and duration (1-1$^1/_2$ hours) of Module Test.
- Clear instruction to candidates.
- Total marks allotted for the Module Test.
- Sub – marks and total marks for each question MUST be clearly indicated.

2. COVERAGE OF THE SYLLABUS CONTENT

- The number of questions in each Module test must be appropriate for the stipulated time of 1-1$^1/_2$hours.
- **CAPE past examination papers should be used as a guide ONLY**.
- Duplication of specific objectives and questions must be avoided.
- Specific objectives tested must be from the relevant Unit of the syllabus.

3. MARK SCHEME

- Detailed mark schemes MUST be submitted, holistic scoring is not recommended, that is, **one mark per skill should be allocated.**
- FRACTIONAL DECIMAL MARKS MUST NOT BE AWARDED.
- The total marks for Module tests MUST be clearly stated on the teacher's solution sheets.
- A student's marks MUST be entered on the FRONT page of the student's script.
- Hand written mark schemes MUST be NEAT and LEGIBLE. The marks MUST be presented in the right hand side of the page.
- Diagrams MUST be neatly drawn with geometrical/mathematical instruments.

4. PRESENTATION OF THE SAMPLE

- Student's responses MUST be written on normal sized paper, preferably 8 $^1/_2$ ×11.
- Question numbers are to be written clearly in the left margin.
- The total marks for each question on students' scripts MUST be clearly written in the right margin.
- ONLY original students' scripts MUST be sent for moderation. **Photocopied scripts will not be accepted.**

- Typed Module tests MUST be in a legible font size (for example, size 12). Hand written texts MUST be NEAT and LEGIBLE.
- The following are required for each Module test:

  - A question paper.
  - Detailed solutions with detailed mark schemes.
  - The scripts (for each Module) of the candidates comprising the sample. The scripts MUST be collated by Modules.

- Marks recorded on the PMath 1 – 3 and PMath 2 – 3 forms must be rounded off to the nearest whole number.
- The guidelines at the bottom of these forms should be observed. (See page 57 of the syllabus, no.6).
- In cases where there are five or more candidates, FIVE samples MUST be sent.
- In cases where there are less than five registered candidates, ALL samples MUST be sent.